The constants λ_1 and λ_2 are expressed in terms of factors of quadratic form Φ and F

$$\lambda_1 = \frac{b_1}{\lambda_{11}}, \quad \lambda_2 = \frac{ab_1 - c_1^2}{\lambda_{11}a}.$$

Here $\lambda_2/\lambda_1 > 1$.

NOTATION

$Q^{(e)}$, external heat flux; Q', uncompensated heat; U, interval energy; S, entropy; T, temperature; $dA^{(i)}$, elementary work of the internal surface forces; η_α, internal degrees of freedom; B_α, thermodynamic parameters conjugate to η_α; Φ, dissipative function; σ_{ij}, stress tensor in the liquid; v_i, liquid-particle velocity; V_0, volume of the porous medium; V, liquid volumes; S_0, surface of volume V_0; S_e, "liquid part" of S_0; S_i, liquid contact surface with the porous matrix inside V_0; P, mean (over S_e) liquid pressure; q_i, filtration velocity; s, Laplace-transform parameters; t, time.

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COMPONENT-TRANSFER EQUATION IN COLUMN WITH LONGITUDINAL SEPARATION

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By the method of "integration over the transverse coordinate," a transfer equation in a separating column in obtained, taking the longitudinal enrichment mechanism into account.

The most widespread type of equipment for isotope separation in gas or liqud phases is a column in which the transverse enrichment effect is converted into a longitudinal effect as a result of circulational flow of the mixture [1]. The derivation of the transfer equation describing the axial distribution of the mean (over the column cross section) concentration of liberated isotope is based on the equilibrium conditions for the component fluxes and the total flux of mixture. To calculate the separation characteristics of equipment of the usual type (thermodiffusional [2, 3] or mass-diffusional [4] columns), it is sufficient to use the transfer equation taking only the radial separation process into account. This kind of relation is obtained, e.g., in [1] by the approximate method of intergration over the transverse coordinate [5], At the same time, in some equipment (in particular, in a separating system with a traveling magnetic wave [6-8]), in addition to the transverse enrichment effect, intensified by forced countercurrent motion of the mixture, there is also a primary longitudinal separation process. There then arises a situation in which circulation has a double effect on the distribution since, on the one hand, it facilitates the multiplication of the transverse effect over the length of the column and, on the other, it causes agitation and, correspondingly, a reduction in the primary longitudinal effect. Below, on the basis of the method of calculations outlined in [1], a transfer equation in a column when both transverse and longitudinal enrichment mechanisms are present is obtained.

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With a view of simplifying the analysis, attention will be confined to separation processes in binary isotopic mixtures. Assuming that the radial component of the hydrodynamic mixture velocity may be neglected, the continuity equation for the heavy component in a cylindrical coordinate system is written in the form

$$\frac{\partial j_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r} rj_r = 0, \tag{1}$$

where

$$j_z = \rho v_z c - \rho D \frac{\partial c}{\partial z} + \varepsilon_z \rho D c (1 - c); \tag{2}$$

$$j_r = -\rho D \frac{\partial c}{\partial r} + \varepsilon_r \rho D c (1 - c); \tag{3}$$

ε_r and ε_z are "local" enrichment factors in the transverse and longitudinal directions. Substituting Eqs. (2) and (3) into Eq. (1), and using the continuity equation for the mixture as a whole, it is found that

$$\rho v_z \frac{\partial c}{\partial z} + \frac{\partial}{\partial z} \varepsilon_z \rho D c (1 - c) - \frac{\partial}{\partial z} \rho D \frac{\partial c}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left\{ r\rho D \left[\frac{\partial c}{\partial r} - \varepsilon_r c (1 - c) \right] \right\} = 0. \tag{4}$$

Taking account of the slight change in concentration c over the radius of the system in Eq. (4) gives

$$\frac{\partial c}{\partial r} = \varepsilon_r c (1 - c) + \left[\frac{1}{\rho D r} \int_0^r \rho v_z r dr \right] \frac{d\bar{c}}{dz} + \frac{d}{dz} \left[\frac{\varepsilon_z \bar{c}(1 - \bar{c})}{\rho D r} \int_0^r \rho D r dr \right] - \left[\frac{1}{\rho D r} \int_0^r r \rho D dr \right] \frac{d^2\bar{c}}{dz^2}. \tag{5}$$

The material-balance equations for the concentrated isotope and the mixture as a whole for the chamber cross section take the form

$$2\pi \int_0^R \left[\rho v_z c r - \rho D r \frac{\partial c}{\partial r} + \varepsilon_z \rho D r c (1 - c) \right] dr = \tau c_\tau, \tag{6}$$

$$2\pi \int_0^R \rho v_z r dr = \tau. \tag{7}$$

Integrating Eq. (6), and using Eqs. (5) and (7), it is found that

$$(K_1 + K_2) \frac{d\bar{c}}{dz} = (K_3 + \varepsilon_z K_2) \bar{c} (1 - \bar{c}) - \frac{d}{dz} [K_4 \varepsilon_z \bar{c} (1 - \bar{c})] - K_4 \frac{d^2\bar{c}}{dz^2} + \tau \bar{c} - \tau c_\tau, \tag{8}$$

where

$$K_1 = 2\pi \int_0^R \frac{\Psi^2}{\rho D r} dr, \quad K_2 = 2\pi \int_0^R \rho D r dr, \quad K_3 = 2\pi \int_0^R \varepsilon_r \Psi dr;$$

$$K_4 = 2\pi \int_0^R \frac{\Psi}{\rho D r} \left[\int_0^r \rho D r dr \right] dr, \quad \Psi = -\int_3^r \rho v_z r dr.$$

When $\varepsilon_z = 0$, which is the case in the absence of the longitudinal separation process, Eq. (8) coincides with the usual mass-transfer equation [1]. Some solutions of Eq. (8) will be obtained. For conditions without withdrawal ($\tau = 0$), when the conditions

$$\frac{D}{R^2} \gg \frac{v_z}{L}, \tag{9}$$

$$\varepsilon_z L \ll 1 \tag{10}$$

are satisfied, Eq. (8) takes its simplest form

$$(K_1 + K_2) \frac{d\bar{c}}{dz} = (K_3 + \varepsilon_z K_2) \bar{c} (1 - \bar{c}). \tag{11}$$

Integrating Eq. (11) from z = 0 to z = L, and taking into account that the longitudinal separation effect is slight, i.e., $(K_3 + \varepsilon_z K_2) L/(K_1 + K_2) \ll 1$, then enrichment factor is found in the form

$$\varepsilon^L = \frac{\left(\frac{\bar{c}}{1-\bar{c}}\right)_L}{\left(\frac{\bar{c}}{1-\bar{c}}\right)_0} - 1 \simeq \int_0^R \frac{K_3 + \varepsilon_z K_2}{K_1 + K_2}\, dz. \tag{12}$$

Assuming that K_1, K_2, and K_3 are constant over the coordinate z, and using, in particular, a relation for ε_z that is satisfied in the case of barodiffusional separation processes in a nonionized gas [9]

$$\varepsilon_z = \alpha_p / p \frac{dp}{dz}, \tag{13}$$

it is found that

$$\varepsilon^L \simeq \frac{K_3 + K_2 \alpha_p \ln \frac{p_L}{p_0}}{K_1 + K_2}. \tag{14}$$

As a specific example, consider a separating system with a traveling magnetic wave [6]. Note that such a system consists of a singular variety of separating columns arranged horizontally [7], in which the radial thermodiffusional effect in the neutral plasma component [10] due to the transverse temperature gradient is multiplied not as a result of thermoconvection in the gravitational field, as in an ordinary thermodiffusion column, but as a result of forced "electromagnetic" plasma convection [6]. Using the relation of [11] with $\eta = \text{const}$ for the circulation-flow intensity, it is found that

$$\varepsilon^L \simeq \frac{\frac{\alpha_t W}{315\pi\eta V_{ph} D} \ln \frac{T_2}{T_1} + \alpha_p \ln \frac{p_L}{p_0}}{1 + \frac{W^2 R^2}{120 \cdot 24^2 \pi^2 D^2 \eta^2 V_{ph}^2 L^2}}. \tag{15}$$

Note that the above-mentioned agitating action of the circulation is described by the second term in the denominator of Eq. (15), and has an identical effect on both the longitudinal and transverse separation effects.

In calculating ε^L, it is necessary to take account of the following circumstance. If the longitudinal pressure gradient is related to the action of inertial or other "mass" forces, the relation $\alpha_p \simeq \Delta\mu/\bar{\mu}$ [9] may be used for constant barodiffusion, where $\Delta\mu$ is the molecular weight difference of the isotope being separated; $\bar{\mu}$ is the mean molecular weight of the mixture. However, in the case when the total pressure gradient of the plasma is due to forces of other origin (e.g., electromagnetic forces, as in the present case), a certain caution is needed in considering the possibility of direct application of Eq. (13). There may arise a situation here such that the separating process occurring in the neutral plasma component is due to diffusional-friction forces acting from the ions "seeping" in the longitudinal direction as a result of ambipolar diffusion [8]. In particular, if the pressure gradient of a binary isotopic mixture is due to frictional forces from ions of a third readily ionized component present in small quantities in the discharge (the situation considered in [12] for a system with crossed fields), the local enrichment factor is given by the relation

$$\varepsilon_z \simeq \frac{\Delta\mu}{2\bar{\mu}} \frac{M}{(M+\bar{\mu})} \left(1 - \frac{Q_{nN}}{Q_{ni}}\right) \frac{1}{p^*} \frac{dp^*}{dz}, \tag{16}$$

where p^* is the neutral gas pressure. As a result, Eq. (15) is found to be valid only with the stipulation that p be understood to mean the pressure of neutrals p^*, and not the total plasma pressure. Then

$$\alpha_p = \frac{\Delta\mu}{2\bar{\mu}} \frac{M}{(M+\bar{\mu})} \left(1 - \frac{Q_{nN}}{Q_{ni}}\right).$$

If one of the components is present in the separation mixture in a small quantity $(\bar{c} \ll 1)$, it is simple to obtain a solution of Eq. (8) in the general case of conditions with withdrawal $(\tau \neq 0)$ as well. Assuming that the feed, with initial concentration c_0, is supplied in the mid cross section of the chamber (z = L/2), and the enriched duct is drawn off at the end (z = L), the enrichment factor ε_+ [11] is found from Eq. (8), taking Eqs. (9) and (10) into account for ε_z = constant, in the form

$$\varepsilon_+ \simeq \frac{\bar{c}_+ - c_0}{c_0} \simeq \frac{\exp\left[\frac{(K_3+\varepsilon_z K_2)L}{2(K_1+K_2)}(1+\lambda)\right]-1}{1+\lambda\exp\left[\frac{(K_3+\varepsilon_z K_2)L}{2(K_1+K_2)}(1+\lambda)\right]}. \quad (17)$$

The separation capacity of the column may be determined using Eq. (17).

Note, in conclusion, that the region of applicability of the results obtained is bounded by the conditions in Eqs. (9) and (10). If it is taken into account that the axial velocity $v_z \simeq v_z^* \simeq D/R$ in the practically most important conditions, when the circulation is optimal for the separation factor, Eq. (9) is valid for $L \gg R$.

NOTATION

j_r, j_z, density of radial and axial diffusional flows of the heavy component; c, heavy-component concentration; $\bar{c}$, mean (over the cross section) concentration; c_0, concentration is feed cross section, $z = L/2$; z, longitudinal coordinate; v_z, axial component of hydrodynamic mixture velocity; v_z^*, velocity corresponding to condition of maximum enrichment factor; Ψ, current function; ρ, mixture density; p, pressure, α_p, α_t, barodiffusional and thermodiffusional constants; T_2, T_1, temperature of hot axial and cold near-wall chamber regions; W, electric power absorbed by gas-discharge plasma; V_{ph}, phase velocity of traveling magnetic wave; η, dynamic viscosity; $\lambda = \tau/(K_3 + \varepsilon_z K_2)$, relative drawoff; L, R, column length and radius; M, ionic mass of readily ionized component; Q_{ni}, Q_{nN}, effective cross section for collisions of atoms of separation mixture with ions and added neutrals, respectively; D, mutual diffusion factor of mixture isotopes.

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